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ABSTRACT

The pamphlet examines the kinds of play that are useful in developing the basis for later abstract thinking--specifically the development of mathematical concepts. The mathematical concepts included are numbers, conservation, one-to-one correspondence, counting and sequencing, ordering, sorting and classification, sorting and sets, and equivalence and nonequivalence. (AS)

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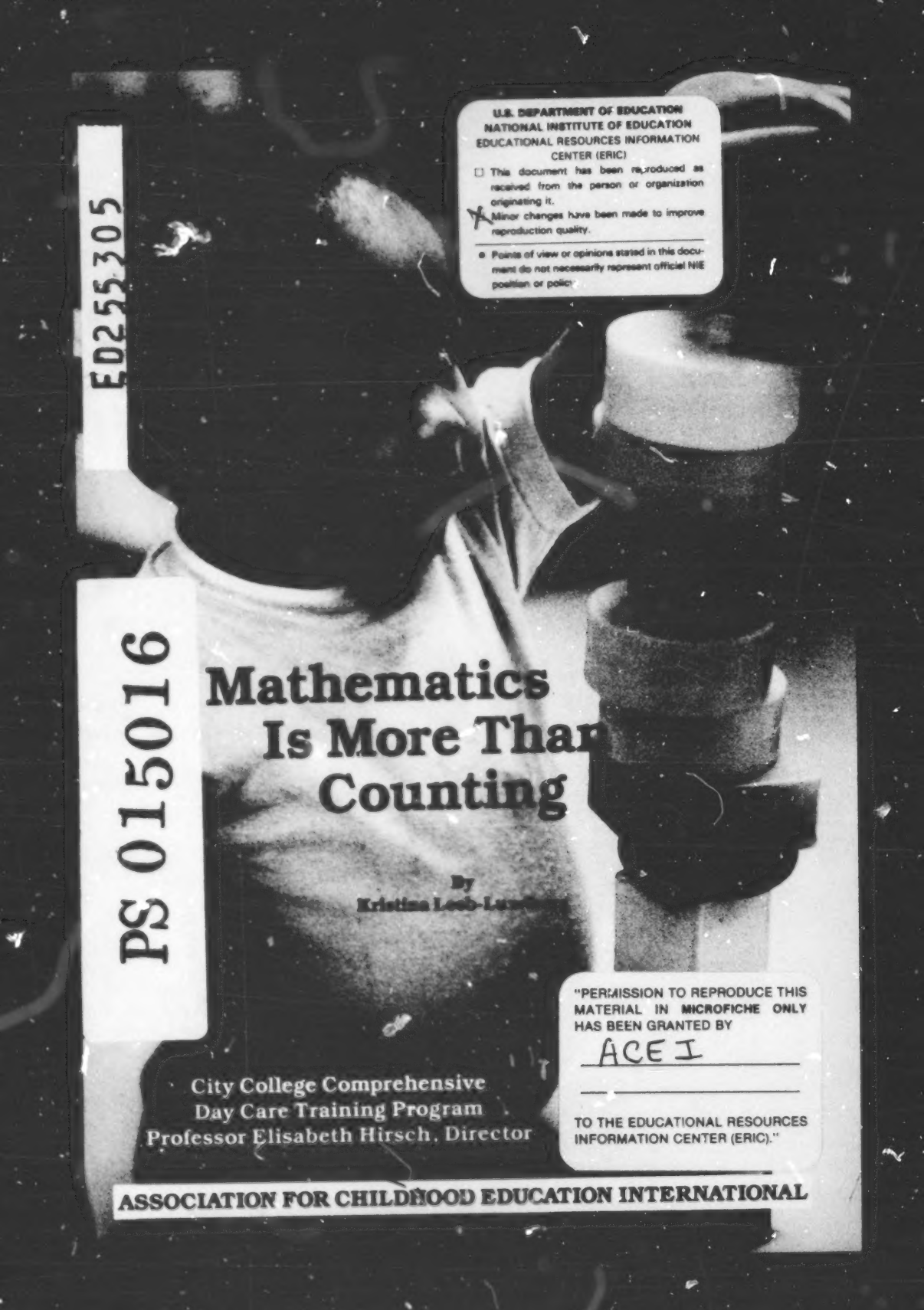
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# Mathematics Is More Than Counting

By
Kristina Loeb-Lu[illegible]

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City College Comprehensive
Day Care Training Program
Professor Elisabeth Hirsch, Director

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Photos by Ann-Marie Mott

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# Mathematics Is More Than Counting

By
**Kristina Leeb-Lundberg**

A majority of normal, bright adults think that mathematics, algebra, geometry—indeed, anything that has to do with numbers—is not their "thing." We now know that this attitude, and the inability that goes along with it, could have been prevented. Abstract thinking (such as doing sums "in your head") should not have been expected of them until the upper elementary grades or later. We also know that real understanding of math can only develop if children have plenty of opportunity to play in ways that will give them a base for later abstract thinking.

Young children can't think only "in their head." They must act, experiment, experience. Children do this naturally when provided with sufficient material and the right environment. We are talking about PLAY.

This pamphlet will examine various kinds of play that are useful in developing the basis for later abstract thinking.

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# I. The Development of Mathematical Concepts



Children are constantly exploring and creating. They are creating when they dig in sand and build with blocks. They are explorers when they go on trips, paint pictures and bake bread. Again and again during these activities, they encounter mathematical ideas and concepts. Through acting on materials they gradually move into mathematics at their own individual speed in their own fashion.

Logical thinking in children develops along with their mathematical discoveries. At the age of two or three they know that one block on top of another is two, and they know that if they add more they will have three, even though they may not know the words *two* and *three*. When they lift objects, they experience lightness or heaviness. Cuddling up in their mother's lap, they feel themselves small and her, big. They know all this by the reality of their experience through living and doing.

They learn with their senses, with their whole bodies. Their understandings become parts of themselves. Only after this has happened can they name these experiences. By the time they learn the words *big, small, light, heavy* or the names of numbers, they will know in their own bodies what these words mean.

Before children come to a day care center, they already have had many other types of experiences with math in their homes and neighborhoods. They may know where the hands of the clock are when they want to watch their favorite TV program. When they want to go to the store to buy ice cream, they might know that they need a quarter or twenty-five



cents. They know they have a choice of the various kinds of things there are to buy. Some children help their parents cook. They watch and perhaps help their parents measure with cups and spoons before putting the ingredients carefully into the bowl for mixing. Children who are allowed to help their parents unpack the family's groceries see and handle many containers of different shapes, size and weights. Their toys, balls, cups and shoes also have various three-dimensional shapes, as do their bed and room. Children experience these properties and come to the center with many pieces of knowledge that they will explore further in their play.

Before coming to the day care center young children will also have had the experience of hearing numbers as names. A child may know the number of his house but think that it is the name of the house. One child knows that he takes the #30 bus to visit his grandmother. When he climbs stairs, his mother will often say, *"one, two, three, four, five, six."* He soon learns that his mother is very pleased when he can recite these numbers himself. But the numbers don't, as yet, have any meaning to him in any mathematical sense. Number words are some of the many things which children pick up by imitation from the adults around them. Yet if you observe the way children use these words, you will see that the number names do not carry the same meaning for them as they do for adults.

As the children come into your classroom you will probably hear children saying things that show their lack of understanding of number words and mathematical concepts. Work and play in different curricu-

lum areas extend and support children's understandings and provide opportunities to clear up their confusions about mathematical concepts.



Cooking allows children to feel "big" and competent while they measure, weigh, count, compare and eat.

Clean up provides opportunities to learn neatness and self-confidence. It also engages children in comparing, sorting, counting and arranging.



Collage and construction provide other experiences in sorting and classifying.



Block building introduces size relat..ons, number quantity, size (without words) and three dimensionality.

Dancing allows children to explore and understand space through their bodies.

## II. Numbers

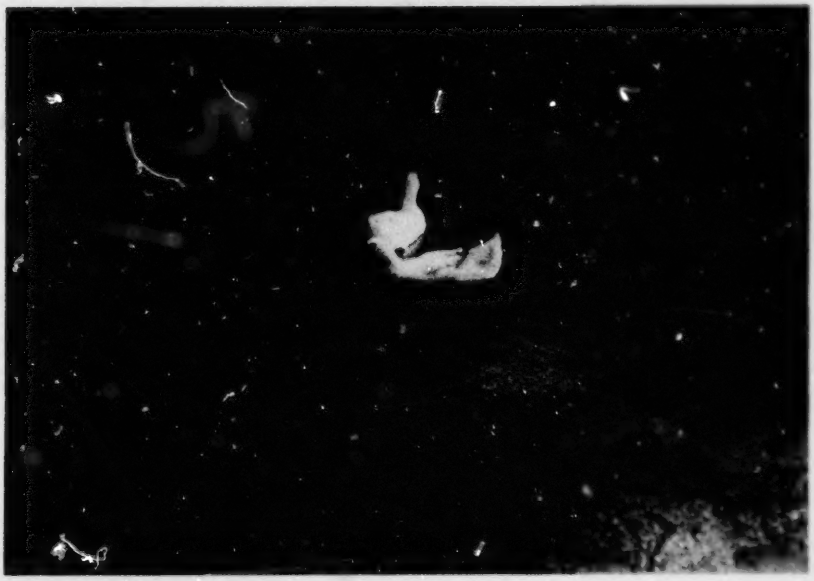
For three year olds their experience is larger than their language. But they have begun to find a match between language and objects. They may know the *twoness* of two because they have two eyes, two hands, two feet, two arms, two legs and so on.

The expansion to three, four and five comes through many experiences of three-, four- and fiveness from manipulation of groups of material objects. The verbalizing and matching of each individual object with a number word becomes counting. Many children understand that the last word counted becomes the number of the whole lot—the set. For other children, the numbers three, four or five have magic qualities—they refer to the child's age!

When three year old Jamal came to the day care center, he already had some number words in his vocabulary and the ability to use them in

some—but by no means all—situations where they belonged. His thinking was dominated by perceptions gained through his senses. In his growing world, he was concerned with and interested in size. When Jamal described himself as a baby, he said, "First, I was this tall" (holding his hand near the floor), " and then I was this tall" (holding his hand near his stomach). "But now, I'm as tall as my head!" This story shows how, like most children his age, Jamal was centered around himself. He was just beginning to understand size through his own body, and this helped him to relate to others in the world.

As a four year old, Jamal's language will expand, but we still may be misled by some of his use of words and perhaps will think he understands more than he actually does. We will have to be close observers and skilled listeners to find out what his statements really mean. He may be likely to describe an object in terms of its action ("A block is to build."), rather than as an adult would—in terms of its properties ("It is made of wood." "It has the shape of a rectangular prism."). This is so, because what matters is still largely related to himself. When it comes to true number and size concepts he probably will still be far from having acquired stable ideas. We often expect children to "know their numbers" before they are generally able to do so. The concept of numbers is the first really abstract notion children learn. However, even if the children know early "how to count" this may not mean that they understand quantity. It takes much longer to grasp *fiveness* than it takes to learn to count to five.



It is helpful to children when teachers use number words in working with them. "Let's put these *two* plants here." "I need *three* more red crayons." Children will soon be able to take in at a glance two or three objects without counting. When this happens the teacher knows that they have attained the concept of *two* or *three*.

Five-year-old Mary was walking through the park with her parents and some of their friends. It was spring, and on the pond a mother duck was swimming with some ducklings behind her. The proud parents, knowing that Mary could count beyond twenty and "knew her numbers," wished to show her ability to their friends. Her father asked her, "How many ducklings are there swimming behind the mother duck?" Mary, fearful and reluctant, was not able to tell her father how many ducklings there were. Her father, seeing her upset, wished to help her and said, "But Mary, you know that the mother duck has five ducklings." Mary burst into tears and said, "There is no one in the middle." Mary's mother and father were confused. They could not understand why Mary could not count the ducklings and decided to speak with Mary's teacher. Monday morning, as they entered the kindergarten classroom, there on the wall was the answer to the puzzle. They saw big cards with domino pictures of numbers from one to ten. This pattern ⚄ represented the number five. This domino pattern of five was the only thing Mary knew about the number five. It became clear to her parents that Mary did not understand the concept of five—the *fiveness* of five in different pattern situations. When the pattern changed, she could no longer recognize five.

It was impossible for Mary to understand that the swimming ducks could also be five because her concept of *fiveness* was not based on her own experiences, but rather on an abstract representation. Mary did not understand that counting "one, two, three, four, five" leads to the same idea as the ⚄ on the classroom wall. Presenting this five year old with an abstract representation did not help her to develop her math understandings. Three and four year olds have an even harder time with this.

Children need a lot of time with various objects in dramatic play, in block building, with table toys experiencing the "fiveness" of five.

When children eventually recognize that five is an abstraction that can mean ····· or ⁘ or ∴, or ✋ they have begun to think in abstractions. Being genuinely able to think about such abstractions is the beginning of true concept formation.

# III. Conservation

Adults are often amazed to realize that children not only think differently from themselves, but that they actually see things differently. For instance, when a little girl is allowed to play with two identical balls of clay ○○ she will agree that they are equal or, if not, she will add to one of them until she is sure that both are the same size. However, if one of these balls is rolled out into a snake in front of her eyes ○⊂⊃ she will insist that the snake is bigger. She will see nothing odd about the snake becoming a ball again and will blithely agree that now it is again the same size as the other ball.

Similar results occur when two identical glasses of liquid are poured, one into another tall thin glass and one into a shallow bowl. A child will probably say that the tall one has more. Or when we have some beads that are close together in a row •••• and the same number of beads in a row, far apart • • • • children will insist that the longer row is "more." Teachers of preschoolers experience this kind of thinking based on perception all the time.



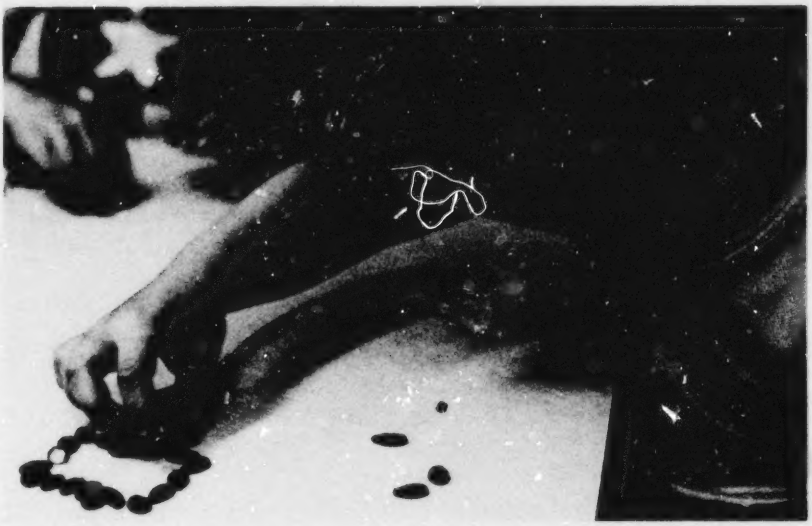
Three four year olds were threading beads one afternoon, each using a string of the same length. Suddenly, Clarence called out, "Look my thread is longer than Sandra's because I got eight on mine and she has only got three." In this case, Clarence mixed up the quantity of beads with the length of the string.

One teacher did an experiment to find out something about what his children really understood about counting. He showed Alice (four years

old) eleven sticks in a row and asked her to count them. Alice did, and said that there were eleven. The teacher then bunched them together and asked Alice to tell him how many there were, without counting. Alice now looked very dubious, although she had seen the whole operation take place. Finally, she said, "I think there are seven." Also, she did not yet understand that the number of sticks was constant, even when they were arranged differently.

What is happening here?

Young children cannot *conserve*. This means that they cannot keep two different qualities in their heads at the same time and that what they *see* wins out. In the case of the clay and the liquid, height or length wins out over quantity (they were unable to conserve quantity "in their heads"). In the case of the sticks, length wins out again over quantity.



This is normal for children at this stage of development. So many young children count without having a real idea of what the number name represents. They cannot yet sort out the physical appearance of the quantity counted related to the actual objects counted. When the objects appear in a new arrangement, the children now look at the space the objects occupy and are unable to keep their attention on the amount. The formation of number concept takes time. Alice's teacher realized that Alice needed many, many more experiences with materials to discover this for herself; that is, in order to develop a true understanding.

Educators have learned from these and similar experiences that children cannot do thinking about sizes or quantities in "their head." They need to act and to experience. The more opportunity they have for

all sorts of firsthand, self-initiated experiences the firmer will be the base upon which their formal math will be built.

Conservation cannot be taught. However, offering many experiences of various kinds with water, sand, cooking, dramatic play, blocks and table toys will help children to understand through experience that things don't become more or less when their appearance changes.

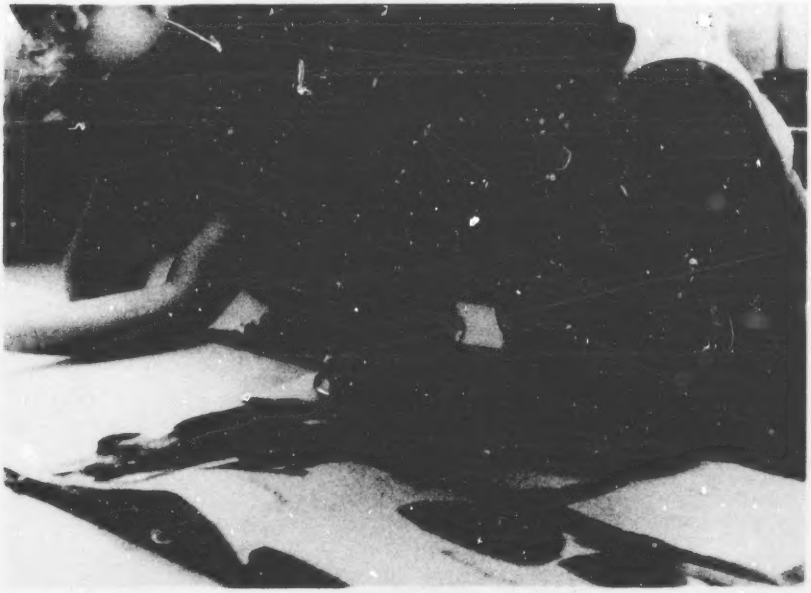
# IV. One-to-One Correspondence

As children explore various materials they will grow to understand that the numbers four, five or ten can represent large objects as well as small ones. Playing with the one-to-one correspondence will help children begin to realize that the towers of the World Trade Center represent the number two, just as well as two lady bugs do. Some things are big and some little, some things are heavy, some are light. But you may have a big thing to go with a little one, or three light objects match three heavy ones. As children play with things that go together, as they experience one-to-one correspondence, they are another step on the way to understanding quantity.

On a sunny winter morning at a day care center in the Bronx, children were helping to set the table for snack. The attentive teacher of the four year olds was aware that matching chairs to napkins to cookies to cups was a fine opportunity for a one-to-one correspondence experience. As the children were setting the table, she prepared twelve chairs and then said, "Put one cookie with each chair." Then, "Let's put one cup each cookie." Now, "How many chairs do we have?" And "How many children are there?" When the children were ready for the playground, she discussed with them the number of coats and hats four children need to put on and the number of hooks they need to hang the coats on. This was an additional experience in one-to-one correspondence. Doing her planning for the following week, she developed an informal graph on what types of pets the children like the most. She would again remember the value of one-to-one correspondence activity for these children. In this way the children learned that four things can be big or they can be little; they can even be PEOPLE.

Practicing one-to-one correspondence between different objects often occurs in unplanned, real-life situations with objects that obviously belong together: mittens to hands, boots to feet, hats to heads and buttons to buttonholes. The understanding of one-to-one correspondence furthers the acquisition of a stable concept of number. Children

need to verify their understanding of one-to-one correspondence through experience with the variety of possibilities that exist. This verification occurs all the time as children explore the numerous activities in their classrooms, their homes and during outdoor play.

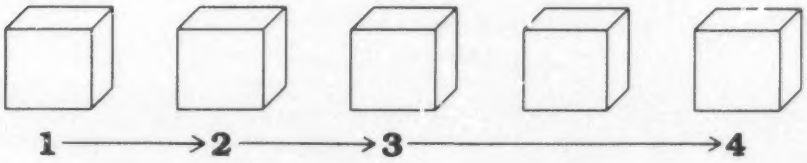


# V. Counting and Sequencing

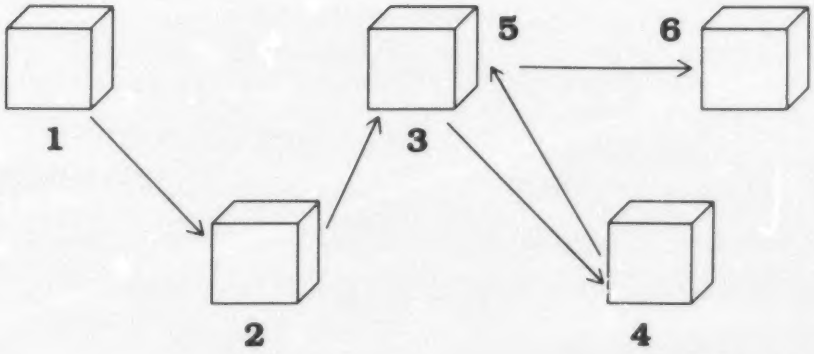
One-to-one correspondence experiences eventually will lead to the ability to match the number words with the actual objects that the children count. For the child to have a clear concept of one-to-one correspondence and counting the teacher must not expect "instant learning." The child's insight at this point may be maturing very slowly.

Counting, an activity many children can do by rote before understanding the meaning of even very small numbers, must acquire true meaning in order to become useful. That children are able to recite numbers in a sequence does not necessarily mean that they understand the number concepts. It may be more like reciting a TV commercial or a popular song. Counting means, really, putting numbers in order; that two is bigger than one; that three is bigger than two and so on.

Counting for a child has many aspects. At the first stages, a child cannot match the number words with the objects counted and may *skip count* or *double count* which results in a wrong total. This is skip counting:




This is double counting:

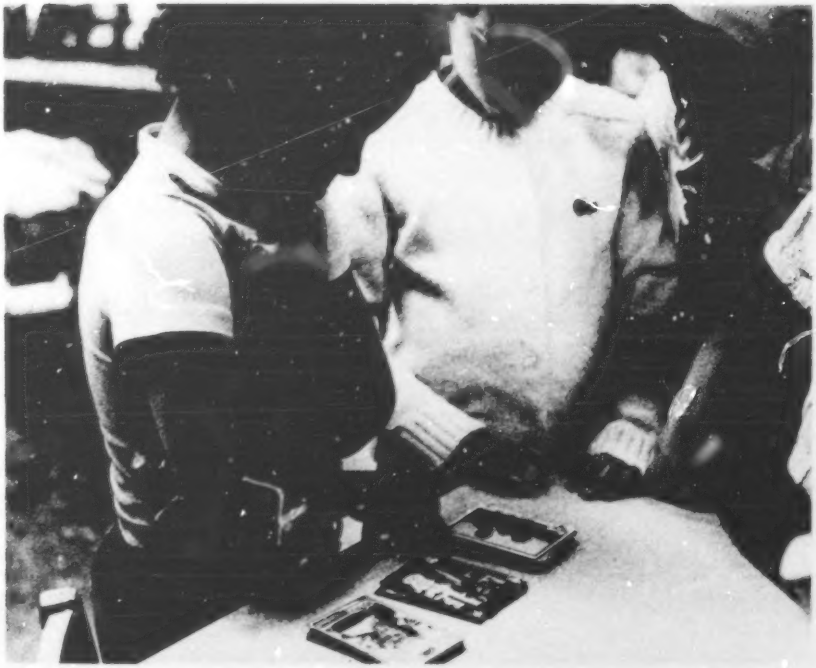



Normally, three-, four- and five-year-olds feel no need to put objects in order so as not to skip or repeat a number.

After children have had activities involving counting, grouping and one-to-one correspondence, the alert teacher should be able to recognize whether their counting ability is rote counting or whether they can match the number words with objects and count with meaning.

Counting will only begin to make sense after children have understood the meaning of numbers (i.e., the fiveness of five) and after they have understood the principle of ordering.

# VI. Ordering



Another mathematical idea that is a vital part of complete number concept formation is the idea of order: Two is more than one, and three is more than two. Number sequence (one, two, three, four) is not the easiest example for young children to start with, since it has the very specific relation of each succeeding number being exactly one more than the preceding number. But before children begin to really understand the relationship of numbers to each other, they must have many concrete experiences of ordering things in different situations in the classroom.

Ordering the environment into series begins when children are very young and continues throughout adult life. The child begins by perceiving opposite ends of a series:

**big — — — — — little**
**heavy — — — — light**
**cold — — — — hot**
**long — — — — short**

The intervention of an adult and suitable material and language lead to refinement of these early basic concepts. The comparison of the height of two children is beginning ordering, as is the comparison of two sets of

things as more or less. Ordering sticks, blocks or nesting cups in a sequence that leads gradually from the smallest to the biggest helps children to see ordered size relations.

When children line up to go out to the playground, they meet another idea of order: Juan stands in front of Kiwesa, and Yvonne stands in back of Lisa. They may use their understanding of sequence when they say, "I want to be first," or "Jimmy is last."

After listening to the story of *The Three Bears*, their observations may become more refined as they discuss the story in a group. The story contains big-little and one-to-one relationships in addition to its many other enjoyable qualities.

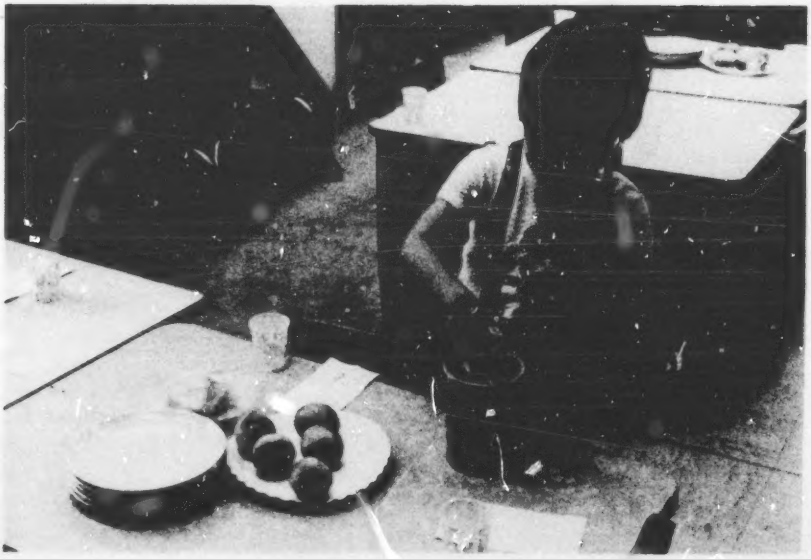
The idea of ordering in size also appears naturally in other classroom areas. The teacher can make ordering a part of natural discussions in relation to the children's play and activities; sets of cans, bottles and books can also be used for practicing ordering of size. With younger children, only two objects are compared at first; this will be extended for older children, to three or more.

Children love to engage in ordering activities and do so spontaneously. Many table toys provide ordering experiences as do ordinary objects like measuring spoons and cups.

The teacher's role is to provide materials and sufficient time. When children come to find the existing materials too easy the teacher can awaken their interest by encouraging them to use the toys differently through asking questions and by providing additional materials.

Ordering activities should include length (sticks), height (bottles), total size (bowls and shoes), weight (stones), color (from light pink to dark red). The possibilities are endless!

# VII. Sorting and Classifying



In order to arrive at a stable concept of number, all preschool children need rich and varied experiences in classification and sorting. Classification and sorting activities are the beginning that helps children to perceive a variety of relationships that are interesting to them. This is how mathematical thinking begins. All mathematical thinking involves relationships of increasing complexity.

The daily life in the classroom contains many opportunities for experiences of sorting. Cleanup is an obvious example. The children bring the things they have played with to their designated places and put them away. They bring the trucks to one corner, hats to the dressup area, books to the library. Occasionally, the teacher uses the mathematical language that the situation naturally gives rise to in order to allow children to learn the words that go with their understandings. "Are *all* the toys put back on the shelf?" "There are still *some* left on the floor, aren't there?" "Oh, you put away *a lot* of things today!" "*How many* books did you put back on the shelf? Can you count them?" Here, counting is included as it now becomes a part of broader experience. Quantitative comparison and ordering, so vital to real concept formation, can happen when children arrange their books in size order.

Mathematics is always with us. It is there as the children sort out what they brought back from their trip to the park, what they collected at the building site or what they brought back from the store. Mathematics is

not isolated from their lives inside or outside school. But the teacher has to make mathematical capital of situations when she feels it comes up in a natural and joyous way.

By providing time and opportunity for children to play with their collections, organize them in various ways, and discuss their reasons for doing so the teacher shows the place of math in our lives. For instance, some children might want to arrange the leaves they collected by size (big and little), others by color, while others still will become aware of the differences between oak, maple and birch leaves.

A teacher of five year olds who found sorting an excellent activity for her class described her approach: "We sort leaves, stems, old shoes, bottle and yogurt tops, colors, shapes. The things we use are endless. I have discovered that sorting can be done in all subject areas. I find that the more I work with math in this way, the more I enjoy it. The children I work with love to do math with me because they get a chance to get on the floor, get out, touch new and strange things. They also get to examine objects they aren't allowed to touch at home. They are beginning to read their experience charts to their parents and to strangers who visit our class." As children sort and classify, they learn by their own experiences the notions of variety and multiplicity. Only when this has been really absorbed are they ready to deal with quantities in a number sense.

# VIII. Sorting and Sets

There are different stages in children's sorting. The youngest children are interested in how the material feels and what they can do with it. Later, they may begin making groups in terms of likeness: all the trucks, all the buttons, all the long sticks. At this stage the teacher does not interfere or give the child any specific direction, but observes what the child does. When the opportunity arises, the teacher should encourage the child to tell him about the sorting. When he feels he can explore the child's readiness for more specific sorting he may ask, "Why did you put these here?" Younger children are often unable to give any specific reason for their choice. This is normal for their age. For instance the teacher may show the child a blue bottlecap and ask, "Can you find another one like it?" The child may select another bottlecap or another blue object without being able to explain the reason for the decision. But through talking with children, the teacher may now help them to put into words the reason why they selected this particular object. The teacher may then suggest that the child make a set (a collection) of

objects that are the same. But the teacher has to allow children to choose what kinds of sets they wish to make. Fours and fives' thinking can be stimulated by specific questions such as: "Can you make piles of different kinds of shells?" "How are they different?" "How are they alike?" "Can you guess which set is smaller?" Old buttons, bottlecaps, corks, shells, screws, leaves, spools, string, popsicle sticks and ribbons are all things that are easy to collect. In sorting, grouping and regrouping, inquisitive children find out how things differ, how some things are small, some large, some red, some blue, some rough, some smooth, some things have a smell, and some things don't have a smell. By sorting all these things according to their various qualities and by discussing what they see, children develop both language and mathematics. They feel, sense and experience in a moving and dynamic way the meaning of a set. A set can consist of many different things at the same time as long as they have one thing in common. Some sets are large (they either have large things in them or more things, if you compare them by matching), some small and some "the same size."

# IX. Equivalence and Nonequivalence



From having practiced one-to-one correspondence with things that belong together, the children are now extending this concept to numerical equivalence (or nonequivalence) between sets with a variety of objects.

Mathematical concepts relating to the ideas of equality and inequality are prerequisites to the idea of number itself. Slowly, children perceive more, less or same amounts in sets, and later, the unchanging *fiveness* or *threeness* of some of these sets. The observing teacher will sense the time this process takes in children, and she will be careful not to push the concepts, but rather to let them grow organically. The same holds true for the language. She has to listen and know how slowly the refinement in language develops. Children use "bigger than" correctly much later than they use "bigger," and they may not achieve this in the preschool years. Only when "bigger" and "smaller" are well understood will they comprehend the idea of "just the same."

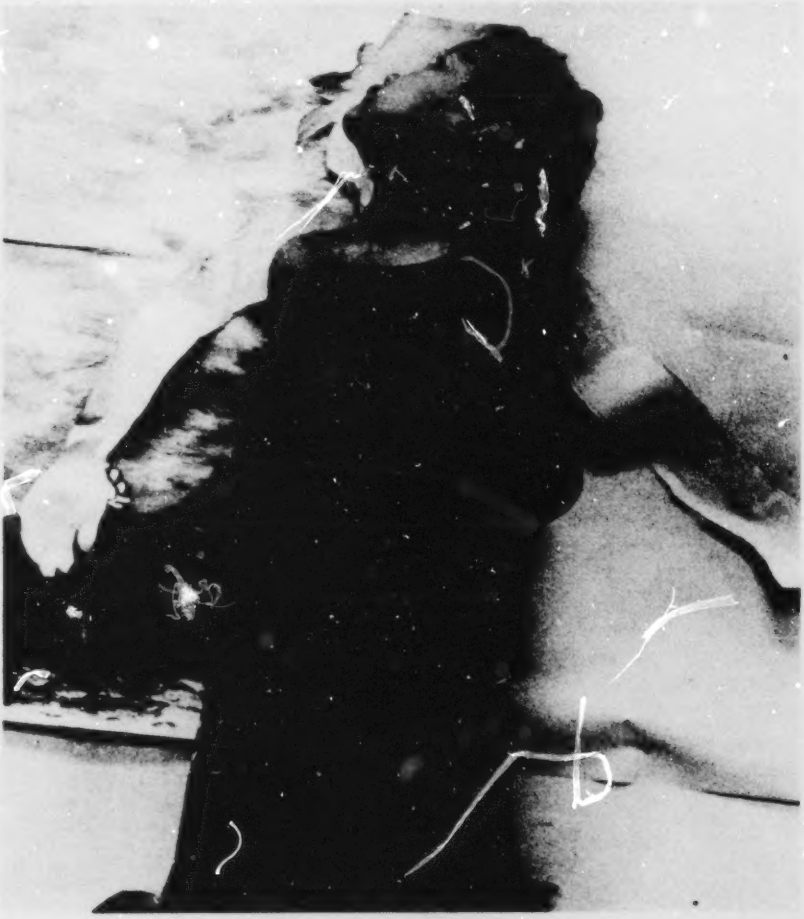
Five year olds enjoy the exploration of equivalence and nonequivalence by experimenting with scales. Scales can be contructed in many ways: with pulleys, baskets hanging from clothes hangers or see-saws.



Mobiles that the children create also provide experiments in nonequivalence. When children use found materials (corks, lids, bits of cloth and paper, clips, sticks) of their own choosing in their mobiles, balance with pulleys and clothes hanger balances, they are actively involved in their own learning and growth.

An active approach has benefits for the teacher, too. As one teacher

commented about her five year olds, "When you work with concrete materials, you find out so much more about the children! I was so surprised. They counted wrong. They counted over and over, most of them one by one. I thought they knew much more than they really did." The teacher learned also that discussion with her children was very valuable. The children would talk about their thinking by discussing the actual sorting experiences. This helped their learning and reasoning.



Some teachers feel strange at first taking their cues from children. As they become involved in and surprised by the freshness, variety and originality of children's ideas they feel that they themselves see the world anew. Teachers can learn and grow with their children.